THE PHILOSOPHY OF LOGICAL ATOMISM.

V. GENERAL PROPOSITIONS AND EXISTENCE.

I AM going to speak to-day about general propositions and existence. The two subjects really belong together; they are the same topic, although it might not have seemed so at the first glance. The propositions and facts that I have been talking about hitherto have all been such as involved only perfectly definite particulars, or relations, or qualities, or things of that sort, never involved the sort of indefinite things one alludes to by such words as "all," "some," "a," "any," and it is propositions and facts of that sort that I am coming on to to-day.

Really all the propositions of the sort that I mean to talk of to-day collect themselves into two groups—the first that are about "all," and the second that are about "some." These two sorts belong together; they are each other's negations. If you say, for instance, "All men are mortal," that is the negative of "Some men are not mortal." In regard to general propositions, the distinction of affirmative and negative is arbitrary. Whether you are going to regard the propositions about "all" as the affirmative ones and the propositions about "some" as the negative ones, or *vice versa*, is purely a matter of taste. For example, if I say "I met no one as I came along," that, on the face of it, you would think is a negative proposition. Of course, that is really a proposition about "all," i. e., "All men are among those whom I did not meet." If, on the other hand, I say "I met a man as I came along," that would strike you

as affirmative, whereas it is the negative of "All men are among those I did not meet as I came along." If you consider such propositions as "All men are mortal" and "Some men are not mortal," you might say it was more natural to take the general propositions as the affirmative and the existence-propositions as the negative, but, simply because it is quite arbitrary which one is to choose, it is better to forget these words and to speak only of general propositions and propositions asserting existence. All general propositions deny the existence of something or other. If you say "All men are mortal," that denies the existence of an immortal man, and so on.

I want to say emphatically that general propositions are to be interpreted as not involving existence. When I say, for instance, "All Greeks are men," I do not want you to suppose that that implies that there are Greeks. It is to be considered emphatically as not implying that. That would have to be added as a separate proposition. If you want to interpret it in that sense, you will have to add the further statement "and there are Greeks." That is for purposes of practical convenience. If you include the fact that there are Greeks, you are rolling two propositions into one, and it causes unnecessary confusion in your logic, because the sorts of propositions that you want are those that do assert the existence of something and general propositions which do not assert existence. If it happened that there were no Greeks, both the proposition that "All Greeks are men" and the proposition that "No Greeks are men" would be true. The proposition "No Greeks are men" is, of course, the proposition "All Greeks are not-men." Both propositions will be true simultaneously if it happens that there are no Greeks. All statements about all the members of a class that has no members are true, because the contradictory of any general statement does assert existence and is therefore false in this case. This

notion, of course, of general propositions not involving existence is one which is not in the traditional doctrine of the syllogism. In the traditional doctrine of the syllogism, it was assumed that when you have such a statement as "All Greeks are men," that implies that there are Greeks, and this produced fallacies. For instance, "All chimeras are animals, and all chimeras breathe flame, therefore some animals breathe flame." This is a syllogism in Darapti, but that mood of the syllogism is fallacious, as this instance shows. That was a point, by the way, which had a certain historical interest, because it impeded Leibniz in his attempts to construct a mathematical logic. He was always engaged in trying to construct such a mathematical logic as we have now, or rather such a one as Boole constructed, and he was always failing because of his respect for Aristotle. Whenever he invented a really good system, as he did several times, it always brought out that such moods as Darapti are fallacious. If you say "All A is B and all A is C, therefore some B is C"—if you say this you incur a fallacy, but he could not bring himself to believe that it was fallacious, so he began again. That shows you that you should not have too much respect for distinguished men.[1]

Now when you come to ask what really is asserted in a general proposition, such as "All Greeks are men" for instance, you find that what is asserted is the truth of all values of what I call a propositional function. A *propositional function* is simply *any expression containing an undetermined constituent, or several undetermined constituents, and becoming a proposition as soon as the undetermined constituents are determined.* If I say "x is a man" or "n is a number," that is a propositional function; so is any formula of algebra, say $(x+y)(x-y) = x^2-y^2$. A propositional function is nothing, but, like most

[1] Cf. Couturat, *La logique de Leibniz.*

of the things one wants to talk about in logic, it does not lose its importance through that fact. The only thing really that you can do with a propositional function is to assert either that it is always true, or that it is sometimes true, or that it is never true. If you take:

"If x is a man, x is mortal,"

that is always true (just as much when x is not a man as when x is a man); if you take:

"x is a man,"

that is sometimes true; if you take:

"x is a unicorn,"

that is never true.

One may call a propositional function

necessary, when it is always true;
possible, when it is sometimes true;
impossible, when it is never true.

Much false philosophy has arisen out of confusing propositional functions and propositions. There is a great deal in ordinary traditional philosophy which consists simply in attributing to propositions the predicates which only apply to propositional functions, and, still worse, sometimes in attributing to individuals predicates which merely apply to propositional functions. This case of *necessary, possible, impossible,* is a case in point. In all traditional philosophy there comes a heading of "modality," which discusses *necessary, possible,* and *impossible* as properties of propositions, whereas in fact they are properties of propositional functions. Propositions are only true or false.

If you take "x is x," that is a propositional function which is true whatever "x" may be, i. e., a necessary propositional function. If you take "x is a man," that is a possible one. If you take "x is a unicorn," that is an impossible one.

Propositions can only be true or false, but propositional functions have these three possibilities. It is important, I think, to realize that the whole doctrine of modality only applies to propositional functions, not to propositions.

Propositional functions are involved in ordinary language in a great many cases where one does not usually realize them. In such a statement as "I met a man," you can understand my statement perfectly well without knowing whom I met, and the actual person is not a constituent of the proposition. You are really asserting there that a certain propositional function is sometimes true, namely the propositional function "I met x and x is human." There is at least one value of x for which that is true, and that therefore is a possible propositional function. Whenever you get such words as "a," "some," "all," "every," it is always a mark of the presence of a propositional function, so that these things are not, so to speak, remote or recondite: they are obvious and familiar.

A propositional function comes in again in such a statement as "Socrates is mortal," because "to be mortal" means "to die at some time or other." You mean there is a time at which Socrates dies, and that again involves a propositional function, namely, that "t is a time, and Socrates dies at t" is possible. If you say "Socrates is immortal," that also will involve a propositional function. That means that "If t is any time whatever, Socrates is alive at time t," if we take immortality as involving existence throughout the whole of the past as well as throughout the whole of the future. But if we take immortality as only involving existence throughout the whole of the future, the interpretation of "Socrates is immortal" becomes more complete, viz., "There is a time t, such that if t' is any time later than t, Socrates is alive at t'." Thus when you come to write out properly what one means by a great many ordinary statements, it turns out a little complicated. "Soc-

rates is mortal" and "Socrates is immortal" are not each other's contradictories, because they both imply that Socrates exists in time, otherwise he would not be either mortal or immortal. One says, "There is a time at which he dies," and the other says, "Whatever time you take, he is alive at that time," whereas the contradictory of "Socrates is mortal" would be true if there is not a time at which he lives.

An undetermined constituent in a propositional function is called a *variable*.

Existence. When you take any propositional function and assert of it that it is possible, that it is sometimes true, that gives you the fundamental meaning of "existence." You may express it by saying that there is at least one value of x for which that propositional function is true. Take "x is a man," there is at least one value of x for which this is true. That is what one means by saying that "There are men," or that "Men exist." Existence is essentially a property of a propositional function. It means that that propositional function is true in at least one instance. If you say "There are unicorns," that will mean that "There is an x, such that x is a unicorn." That is written in phrasing which is unduly approximated to ordinary language, but the proper way to put it would be "(x is a unicorn) is possible." We have got to have some idea that we do not define, and one takes the idea of "always true," or of "sometimes true," as one's undefined idea in this matter, and then you can define the other one as the negative of that. In some ways it is better to take them both as undefined, for reasons which I shall not go into at present. It will be out of this notion of *sometimes*, which is the same as the notion of *possible*, that we get the notion of existence. To say that unicorns exist is simply to say that "(x is a unicorn) is possible."

It is perfectly clear that when you say "Unicorns exist,"

you are not saying anything that would apply to any unicorns there might happen to be, because as a matter of fact there are not any, and therefore if what you say had any application to the actual individuals, it could not possibly be significant unless it were true. You can consider the proposition "Unicorns exist" and can see that it is false. It is not nonsense. Of course, if the proposition went through the general conception of the unicorn to the individual, it could not be even significant unless there were unicorns. Therefore when you say "Unicorns exist," you are not saying anything about any individual things, and the same applies when you say "Men exist." If you say that "Men exist, and Socrates is a man, therefore Socrates exists," that is exactly the same sort of fallacy as it would be if you said "Men are numerous, Socrates is a man, therefore Socrates is numerous," because existence is a predicate of a propositional function, or derivatively of a class. When you say of a propositional function that it is numerous, you will mean that there are several values of x that will satisfy it, that there are more than one; or, if you like to take "numerous" in a larger sense, more than ten, more than twenty, or whatever number you think fitting. If x, y, and z all satisfy a propositional function, you may say that that proposition is numerous, but x, y, and z severally are not numerous. Exactly the same applies to existence, that is to say that the actual things that there are in the world do not exist, or, at least, that is putting it too strongly, because that is utter nonsense. To say that they do not exist is strictly nonsense, but to say that they do exist is also strictly nonsense.

It is of propositional functions that you can assert or deny existence. You must not run away with the idea that this entails consequences that it does not entail. If I say "The things that there are in the world exist," that

is a perfectly correct statement, because I am there saying something about a certain class of things; I say it in the same sense in which I say "Men exist." But I must not go on to "This is a thing in the world, and therefore this exists." It is there the fallacy comes in, and it is simply, as you see, a fallacy of transferring to the individual that satisfies a propositional function a predicate which only applies to a propositional function. You can see this in various ways. For instance, you sometimes know the truth of an existence-proposition without knowing any instance of it. You know that there are people in Timbuctoo, but I doubt if any of you could give me an instance of one. Therefore you clearly can know existence-propositions without knowing any individual that makes them true. Existence-propositions do not say anything about the actual individual but only about the class or function.

It is exceedingly difficult to make this point clear as long as one adheres to ordinary language, because ordinary language is rooted in a certain feeling about logic, a certain feeling that our primeval ancestors had, and as long as you keep to ordinary language you find it very difficult to get away from the bias which is imposed upon you by language. When I say, e. g., "There is an x such that x is a man," that is not the sort of phrase one would like to use. "There is an x" is meaningless. What is "an x" anyhow? There is not such a thing. The only way you can really state it correctly is by inventing a new language *ad hoc,* and making the statement apply straight off to "x is a man," as when one says "(x is a man) is possible," or invent a special symbol for the statement that "x is a man" is sometimes true.

I have dwelt on this point because it really is of very fundamental importance. I shall come back to existence in my next lecture: existence as it applies to descriptions, which is a slightly more complicated case than I am dis-

cussing here. I think an almost unbelievable amount of false philosophy has arisen through not realizing what "existence" means.

As I was saying a moment ago, a propositional function in itself is nothing: it is merely a schema. Therefore in the inventory of the world, which is what I am trying to get at, one comes to the question, What is there really in the world that corresponds with these things? Of course, it is clear that we have general *propositions,* in the same sense in which we have atomic propositions. For the moment I will include existence-propositions with general propositions. We have such propositions as "All men are mortal" and "Some men are Greeks." But you have not only such *propositions*; you have also such *facts*, and that, of course, is where you get back to the inventory of the world: that, in addition to particular facts, which I have been talking about in previous lectures, there are also general facts and existence-facts, that is to say, there are not merely *propositions* of that sort but also *facts* of that sort. That is rather an important point to realize. You cannot ever arrive at a general fact by inference from particular facts, however numerous. The old plan of complete induction, which used to occur in books, which was always supposed to be quite safe and easy as opposed to ordinary induction, that plan of complete induction, unless it is accompanied by at least one general proposition, will not yield you the result that you want. Suppose, for example, that you wish to prove in that way that "All men are mortal," you are supposed to proceed by complete induction, and say "A is a man that is mortal," "B is a man that is mortal," "C is a man that is mortal," and so on until you finish. You will not be able, in that way, to arrive at the proposition "All men are mortal" unless you know when you have finished. That is to say that, in order to arrive by this road at the general proposition "All men are mortal," you

must already have the general proposition "All men are among those I have enumerated." You never can arrive at a general proposition by inference from particular propositions alone. You will always have to have at least one general proposition in your premises. That illustrates, I think, various points. One, which is epistemological, is that if there is, as there seems to be, knowledge of general propositions, then there must be *primitive* knowledge of general propositions (I mean by that, knowledge of general propositions which is not obtained by inference), because if you can never infer a general proposition except from premises of which one at least is general, it is clear that you can never have knowledge of such propositions by inference unless there is knowledge of some general propositions which is not by inference. I think that the sort of way such knowledge—or rather the belief that we have such knowledge—comes into ordinary life is probably very odd. I mean to say that we do habitually assume general propositions which are exceedingly doubtful; as, for instance, one might, if one were counting up the people in this room, assume that one could see all of them, which is a general proposition, and very doubtful as there may be people under the tables. But, apart from that sort of thing, you do have in any empirical verification of general propositions some kind of assumption that amounts to this, that what you do not see is not there. Of course, you would not put it so strongly as that, but you would assume that, with certain limitations and certain qualifications, if a thing does not appear to your senses, it is not there. That is a general proposition, and it is only through such propositions that you arrive at the ordinary empirical results that one obtains in ordinary ways. If you take a census of the country, for instance, you assume that the people you do not see are not there, provided you search properly and carefully, otherwise your census might be wrong. It

is some assumption of that sort which would underlie what seems purely empirical. You could not prove empirically that what you do not perceive is not there, because an empirical proof would consist in perceiving, and by hypothesis you do not perceive it, so that any proposition of that sort, if it is accepted, has to be accepted on its own evidence. I only take that as an illustration. There are many other illustrations one could take of the sort of propositions that are commonly assumed, many of them with very little justification.

I come now to a question which concerns logic more nearly, namely, the reasons for supposing that there are general facts as well as general propositions. When we were discussing molecular propositions I threw doubt upon the supposition that there are molecular facts, but I do not think one can doubt that there are general facts. It is perfectly clear, I think, that when you have enumerated all the atomic facts in the world, it is a further fact about the world that those are all the atomic facts there are about the world, and that is just as much an objective fact about the world as any of them are. It is clear, I think, that you must admit general facts as distinct from and over and above particular facts. The same thing applies to "All men are mortal." When you have taken all the particular men that there are, and found each one of them severally to be mortal, it is definitely a new fact that all men are mortal; how new a fact, appears from what I said a moment ago, that it could not be inferred from the mortality of the several men that there are in the world. Of course, it is not so difficult to admit what I might call existence-facts—such facts as "There are men," "There are sheep," and so on. Those, I think, you will readily admit as separate and distinct facts over and above the atomic facts I spoke of before. Those facts have got to come into the inventory of the world, and in that way

propositional functions come in as involved in the study of general facts. I do not profess to know what the right analysis of general facts is. It is an exceedingly difficult question, and one which I should very much like to see studied. I am sure that, although the convenient technical treatment is by means of propositional functions, that is not the whole of the right analysis. Beyond that I cannot go.

There is one point about whether there are molecular facts. I think I mentioned, when I was saying that I did not think there were disjunctive facts, that a certain difficulty does arise in regard to general facts. Take "All men are mortal." That means:

" 'x is a man' implies
'x is a mortal' whatever
x may be."

You can see at once that it is a hypothetical proposition. It does not imply that there are any men, nor who are men, and who are not; it simply says that if you have anything which is a man, that thing is mortal. As Mr. Bradley has pointed out in the second chapter of his *Principles of Logic,* "Trespassers will be prosecuted" may be true even if no one trespasses, since it means merely that, *if* any one trespasses, he will be prosecuted. It comes down to this that

" 'x is a man' implies 'x is a mortal'
is always true,"

is a fact. It is perhaps a little difficult to see how that can be true if one is going to say that " 'Socrates is a man' implies 'Socrates is a mortal' " is not itself a fact, which is what I suggested when I was discussing disjunctive facts. I do not feel sure that you could not get round that difficulty. I only suggest it as a point which should be considered when one is denying that there are molecular

facts, since, if it cannot be got round, we shall have to admit molecular facts.

Now I want to come to the subject of *completely general* propositions and propositional functions. By those I mean propositions and propositional functions that contain only variables and nothing else at all. This covers the whole of logic. Every logical proposition consists wholly and solely of variables, though it is not true that every proposition consisting wholly and solely of variables is logical. You can consider stages of generalizations as, e. g.,

"Socrates loves Plato"
"x loves Plato"
"x loves y"
"x R y."

There you have been going through a process of successive generalization. When you have got to $x\mathrm{R}y$, you have got a schema consisting only of variables, containing no constants at all, the pure schema of dual relations, and it is clear that any proposition which expresses a dual relation can be derived from $x\mathrm{R}y$ by assigning values to x and R and y. So that that is, as you might say, the pure form of all those propositions. I mean by the form of a proposition that which you get when for every single one of its constituents you substitute a variable. If you want a different definition of the form of a proposition, you might be inclined to define it as the class of all those propositions that you can obtain from a given one by substituting other constituents for one or more of the constituents the proposition contains. E. g., in "Socrates loves Plato," you can substitute somebody else for Socrates, somebody else for Plato, and some other verb for "loves." In that way there are a certain number of propositions which you can derive from the proposition "Socrates loves Plato," by replacing the constituents of that proposition by other constituents,

so that you have there a certain class of propositions, and those propositions all have a certain form, and one can, if one likes, say that the form they all have is the class consisting of all of them. That is rather a provisional definition, because as a matter of fact, the idea of form is more fundamental than the idea of class. I should not suggest that as a really good definition, but it will do provisionally to explain the sort of thing one means by the form of a proposition. The form of a proposition is that which is in common between any two propositions of which the one can be obtained from the other by substituting other constituents for the original ones. When you have got down to those formulas that contain only variables, like $x\mathrm{R}y$, you are on the way to the sort of thing that you can assert in logic.

To give an illustration, you know what I mean by the domain of a relation: I mean all the terms that have that relation to something. Suppose I say: "$x\mathrm{R}y$ implies that x belongs to the domain of R," that would be a proposition of logic and is one that contains only variables. You might think it contains such words as "belong" and "domain," but that is an error. It is only the habit of using ordinary language that makes those words appear. They are not really there. That is a proposition of pure logic. It does not mention any particular thing at all. This is to be understood as being asserted whatever x and R and y may be. All the statements of logic are of that sort.

It is not a very easy thing to see what are the constituents of a logical proposition. When one takes "Socrates loves Plato," "Socrates" is a constituent, "loves" is a constituent, and "Plato" is a constituent. Then you turn "Socrates" into x, "loves" into R, and "Plato" into y. x and R and y are nothing, and they are not constituents, so it seems as though all the propositions of logic were entirely devoid of constituents. I do not think that can

quite be true. But then the only other thing you can seem to say is that the *form* is a constituent, that propositions of a certain form are always true: that *may* be the right analysis, though I very much doubt whether it is.

There is, however, just this to observe, viz., that the form of a proposition is never a constituent of that proposition itself. If you assert that "Socrates loves Plato," the form of that proposition is the form of the dual relation, but this is not a constituent of the proposition. If it were you would have to have that constituent related to the other constituents. You will make the form much too substantial if you think of it as really one of the things that have that form, so that the form of a proposition is certainly not a constituent of the proposition itself. Nevertheless it may possibly be a constituent of general statements about propositions that have that form, so I think it is *possible that* logical propositions might be interpreted as being about forms.

I can only say, in conclusion, as regards the constituents of logical propositions, that it is a problem which is rather new. There has not been much opportunity to consider it. I do not think any literature exists at all which deals with it in any way whatever, and it is an interesting problem.

I just want now to give you a few illustrations of propositions which can be expressed in the language of pure variables but are not propositions of logic. Among the propositions that are propositions of logic are included all the propositions of pure mathematics, all of which cannot only be expressed in logical terms but can also be deduced from the premises of logic, and therefore they are logical propositions. Apart from them there are many that can be expressed in logical terms, but cannot be proved from logic, and are certainly not propositions that form part of logic. Suppose you take such a proposition as: "There is at least one thing in the world." That is a proposition that you

can express in logical terms. It will mean, if you like, that the propositional function "$x = x$" is a possible one. That is a proposition, therefore, that you can express in logical terms; but you cannot know from logic whether it is true or false. So far as you do know it, you know it empirically, because there might happen not to be a universe, and then it would not be true. It is merely an accident, so to speak, that there is a universe. The proposition that there are exactly 30,000 things in the world can also be expressed in purely logical terms, and is certainly not a proposition of logic but an empirical proposition (true or false), because a world containing more than 30,000 things and a world containing fewer than 30,000 things are both possible, so that if it happens that there are exactly 30,000 things, that is what one might call an accident and is not a proposition of logic. There are again two propositions that one is used to in mathematical logic, namely, the multiplicative axiom and the axiom of infinity. These also can be expressed in logical terms, but cannot be proved or disproved by logic. In regard to the axiom of infinity, the impossibility of logical proof or disproof may be taken as certain, but in the case of the multiplicative axiom, it is perhaps still open to some degree to doubt. Everything that is a proposition of logic has got to be in some sense or other like a tautology. It has got to be something that has some peculiar quality, which I do not know how to define, that belongs to logical propositions and not to others. Examples of typical logical propositions are:

"If p implies q and q implies r, then p implies r."

"If all a's are b's and all b's are c's, then all a's are c's.

"If all a's are b's, and x is an a, then x is a b."

Those are propositions of logic. They have a certain peculiar quality which marks them out from other propositions and enables us to know them *a priori*. But what exactly that characteristic is, I am not able to tell you. Although it is a necessary characteristic of logical propositions that they should consist solely of variables, i. e., that they should assert the universal truth, or the sometimes-truth, of a propositional function consisting wholly of variables—although that is a necessary characteristic, it is not a sufficient one.

I am sorry that I have had to leave so many problems unsolved. I always have to make this apology, but the world really is rather puzzling and I cannot help it.

DISCUSSION.

.....Is there any word you would substitute for "existence" which would give existence to individuals? Are you applying the word "existence" to two ideas, or do you deny that there are two ideas?

Mr. Russell: No, there is not an idea that will apply to individuals. As regards the actual things there are in the world, there is nothing at all you can say about them that in any way corresponds to this notion of existence. It is a sheer mistake to say that there is anything analogous to existence that you can say about them. You get into confusion through language, because it is a perfectly correct thing to say "All the things in the world exist," and it is so easy to pass from this to "This exists because it is a thing in the world." There is no sort of point in a predicate which could not conceivably be false. I mean, it is perfectly clear that, if there were such a thing as this existence of individuals that we talk of, it would be absolutely impossible for it not to apply, and that is the characteristic of a mistake.

VI. DESCRIPTIONS AND INCOMPLETE SYMBOLS.

I am proposing to deal this time with the subject of descriptions, and what I call "incomplete symbols," and

the existence of described individuals. You will remember that last time I dealt with the existence of *kinds* of things, what you mean by saying "There are men" or "There are Greeks" or phrases of that sort, where you have an existence which may be plural. I am going to deal to-day with an existence which is asserted to be singular, such as "The man with the iron mask existed" or some phrase of that sort, where you have some object described by the phrase "The so-and-so" in the singular, and I want to discuss the analysis of propositions in which phrases of that kind occur.

There are, of course, a great many propositions very familiar in metaphysics which are of that sort: "I exist" or "God exists" or "Homer existed," and other such statements are always occurring in metaphysical discussions, and are, I think, treated in ordinary metaphysics in a way which embodies a simple logical mistake that we shall be concerned with to-day, the same sort of mistake that I spoke of last week in connection with the existence of kinds of things. One way of examining a proposition of that sort is to ask yourself what would happen if it were false. If you take such a proposition as "Romulus existed," probably most of us think that Romulus did not exist. It is obviously a perfectly significant statement, whether true or false, to say that Romulus existed. If Romulus himself entered into our statement, it would be plain that the statement that he did not exist would be nonsense, because you cannot have a constituent of a propositon which is nothing at all. Every constituent has got to be there as one of the things in the world, and therefore if Romulus himself entered into the propositions that he existed or that he did not exist, both these propositions could not only not be true, but could not be even significant, unless he existed. That is obviously not the case, and the first conclusion one draws is that, although it *looks* as if Romulus were a constituent

of that proposition, that is really a mistake. Romulus does not occur in the proposition "Romulus did not exist."

Suppose you try to make out what you do mean by that proposition. You can take, say, all the things that Livy has to say about Romulus, all the properties he ascribes to him, including the only one probably that most of us remember, namely, the fact that he was called "Romulus." You can put all this together, and make a propositional function saying "x has such-and-such properties," the properties being those you find enumerated in Livy. There you have a propositional function, and when you say that Romulus did not exist you are simply saying that that propositional function is never true, that it is impossible in the sense I was explaining last time, i. e., that there is no value of x that makes it true. That reduces the non-existence of Romulus to the sort of non-existence I spoke of last time, where we had the non-existence of unicorns. But it is not a *complete* account of this kind of existence or non-existence, because there is one other way in which a described individual can fail to exist, and that is where the description applies to more than one person. You cannot, e. g., speak of "*The* inhabitant of London," not because there are none, but because there are so many.

You see, therefore, that this proposition "Romulus existed" or "Romulus did not exist" does introduce a propositional function, because the name "Romulus" is not really a name but a sort of truncated description. It stands for a person who did such-and-such things, who killed Remus, and founded Rome, and so on. It is short for that description; if you like, it is short for "the person who was called 'Romulus.'" If it were really a name, the question of existence could not arise, because a name has got to name something or it is not a name, and if there is no such person as Romulus there cannot be a name for that person who is not there, so that this single word "Romulus" is really

a sort of truncated or telescoped description, and if you think of it as a name you will get into logical errors. When you realize that it is a description, you realize therefore that any proposition about Romulus really introduces the propositional function embodying the description, as (say) "x was called 'Romulus.'" That introduces you at once to a propositional function, and when you say "Romulus did not exist," you mean that this propositional function is not true for one value of x.

There are two sorts of descriptions, what one may call "ambiguous descriptions," when we speak of "*a* so-and-so," and what one may call "definite descriptions," when we speak of "*the* so-and-so" (in the singular). Instances are:

Ambiguous: A man, a dog, a pig, a Cabinet Minister.

Definite: The man with the iron mask.
The last person who came into this room.
The only Englishman who ever occupied the Papal See.
The number of the inhabitants of London.
The sum of 43 and 34.

(It is not necessary for a description that it should describe an individual: it may describe a predicate or a relation or anything else.)

It is phrases of that sort, definite descriptions, that I want to talk about to-day. I do not want to talk about ambiguous descriptions, as what there was to say about them was said last time.

I want you to realize that the question whether a phrase is a definite description turns only upon its form, not upon the question whether there is a definite individual so described. For instance, I should call "The inhabitant of London" a definite description, although it does not in fact describe any definite individual.

The first thing to realize about a definite description

is that it is not a name. We will take "The author of *Waverley*." That is a definite description, and it is easy to see that it is not a name. A name is a simple symbol (i. e., a symbol which does not have any parts that are symbols), a simple symbol used to designate a certain particular or by extension an object which is not a particular but is treated for the moment as if it were, or is falsely believed to be a particular, such as a person. This sort of phrase, "The author of *Waverley*," is not a name because it is a complex symbol. It contains parts which *are* symbols. It contains four words, and the meanings of those four words are already fixed and they have fixed the meaning of "The author of *Waverley*" in the only sense in which that phrase does have any meaning. In that sense, its meaning is already determinate, i. e., there is nothing arbitrary or conventional about the meaning of that whole phrase, when the meanings of "the," "author," "of," and "*Waverley*" have already been fixed. In that respect, it differs from "Scott," because when you have fixed the meaning of all the other words in the language, you have done nothing toward fixing the meaning of the name "Scott." That is to say, if you understand the English language, you would understand the meaning of the phrase "The author of *Waverley*" if you had never heard it before, whereas you would not understand the meaning of "Scott" if you had never heard the word before because to know the meaning of a name is to know who it is applied to.

You sometimes find people speaking as if descriptive phrases were names, and you will find it suggested, e. g., that such a proposition as "Scott is the author of *Waverley*" really asserts that "Scott" and "the author of *Waverley*" are two names for the same person. That is an entire delusion; first of all, because "the author of *Waverley*" is not a name, and, secondly, because, as you can perfectly

well see, if that were what is meant, the proposition would be one like "Scott is Sir Walter," and would not depend upon any fact except that the person in question was so called, because a name is what a man is called. As a matter of fact, Scott was the author of *Waverley* at a time when no one called him so, when no one knew whether he was or not, and the fact that he was the author was a physical fact, the fact that he sat down and wrote it with his own hand, which does not have anything to do with what he was called. It is in no way arbitrary. You cannot settle by any choice of nomenclature whether he is or is not to be the author of *Waverley,* because in actual fact he chose to write it and you cannot help yourself. That illustrates how "the author of *Waverley*" is quite a different thing from a name. You can prove this point very clearly by formal arguments. In "Scott is the author of *Waverley*" the "is," of course, expresses identity, i. e., the entity whose name is Scott is identical with the author of *Waverley*. But, when I say "Scott is mortal" this "is" is the "is" of predication, which is quite different from the "is" of identity. It is a mistake to interpret "Scott is mortal" as meaning "Scott is identical with one among mortals," because (among other reasons) you will not be able to say what "mortals" are except by means of the propositional function "x is mortal," which brings back the "is" of predication. You cannot reduce the "is" of predication to the other "is." But the "is" in "Scott is the author of *Waverley*" is the "is" of identity and not of predication.[1]

If you were to try to substitute for "the author of *Waverley*" in that proposition any name whatever, say "c," so that the proposition becomes "Scott is c," then if "c" is a name for anybody who is not Scott, that proposition

[1] The confusion of these two meanings of "is" is essential to the Hegelian conception of identity-in-difference.

would become false, while if, on the other hand, "*c*" is a name for Scott, then the proposition will become simply a tautology. It is at once obvious that if "*c*" were "Scott" itself, "Scott is Scott" is just a tautology. But if you take any other name which is just a name for Scott, then if the name is being used *as* a name and not as a description, the proposition will still be a tautology. For the name itself is merely a means of pointing to the thing, and does not occur in what you are asserting, so that if one thing has two names, you make exactly the same assertion whichever of the two names you use, provided they are really names and not truncated descriptions.

So there are only two alternatives. If "*c*" is a name, the proposition "Scott is *c*" is either false or tautologous. But the proposition "Scott is the author of *Waverley*" is neither, and therefore is not the same as any proposition of the form "Scott is *c*," where "*c*" is a name. That is another way of illustrating the fact that a description is quite a different thing from a name.

I should like to make clear what I was saying just now, that if you substitute another name in place of "Scott" which is also a name of the same individual, say, "Scott is Sir Walter," then "Scott" and "Sir Walter" are being used as names and not as descriptions, your proposition is strictly a tautology. If one asserts "Scott is Sir Walter," the way one would mean it would be that one was using the names as descriptions. One would mean that the person called "Scott" is the person called "Sir Walter," and "the person called 'Scott' " is a description, and so is "the person called 'Sir Walter.' " So that would not be a tautology. It would mean that the person called "Scott" is identical with the person called "Sir Walter." But if you are using both as names, the matter is quite different. You must observe that the name does not occur in that which you assert when you use the name. The name is

merely that which is a means of expressing what it is you are trying to assert, and when I say "Scott wrote *Waverley*," the name "Scott" does not occur in the thing I am asserting. The thing I am asserting is about the person, not about the name. So if I say "Scott is Sir Walter," using these two names *as* names, neither "Scott" nor "Sir Walter" occurs in what I am asserting, but only the person who has these names, and thus what I am asserting is a pure tautology.

It is rather important to realize this about the two different uses of names or of any other symbols: the one when you are talking about the symbol and the other when you are using it *as* a symbol, as a means of talking about something else. Normally, if you talk about your dinner, you are not talking about the word "dinner" but about what you are going to eat, and that is a different thing altogether. The ordinary use of words is as a means of getting through to things, and when you are using words in that way the statement "Scott is Sir Walter" is a pure tautology, exactly on the same level as "Scott is Scott."

That brings me back to the point that when you take "Scott is the author of *Waverley*" and you substitute for "the author of *Waverley*" a name in the place of a description, you get necessarily either a tautology or a falsehood—a tautology if you substitute "Scott" or some other name for the same person, and a falsehood if you substitute anything else. But the proposition itself is neither a tautology nor a falsehood, and that shows you that the proposition "Scott is the author of *Waverley*" is a different proposition from any that can be obtained if you substitute a name in the place of "the author of *Waverley*." That conclusion is equally true of any other proposition in which the phrase "the author of *Waverley*" occurs. If you take any proposition in which that phrase occurs and substitute for that phrase a proper name, whether that name be

"Scott" or any other, you will get a different proposition. Generally speaking, if the name that you substitute is "Scott," your proposition, if it was true before will remain true, and if it was false before will remain false. But it is a *different* proposition. It is not *always* true that it will remain true or false, as may be seen by the example: "George IV wished to know if Scott was the author of *Waverley*." It is not true that George IV wished to know if Scott was Scott. So it is even the case that the truth or the falsehood of a proposition is sometimes changed when you substitute a name of an object for a description of the same object. But in any case it is always a different proposition when you substitute a name for a description.

Identity is a rather puzzling thing at first sight. When you say "Scott is the author of *Waverley*," you are half-tempted to think there are two people, one of whom is Scott and the other the author of *Waverley*, and they happen to be the same. That is obviously absurd, but that is the sort of way one is always tempted to deal with identity.

When I say "Scott is the author of *Waverley*" and that "is" expresses identity, the reason that identity can be asserted there truly and without tautology is just the fact that the one is a name and the other a description. Or they might both be descriptions. If I say "The author of *Waverley* is the author of *Marmion*," that, of course, asserts identity between two descriptions.

Now the next point that I want to make clear is that when a description (when I say "description" I mean, for the future, a *definite* description) occurs in a proposition, there is no constituent of that proposition corresponding to that description as a whole. In the true analysis of the proposition, the description is broken up and disappears. That is to say, when I say "Scott is the author of *Waverley*" it is a wrong analysis of that to suppose that you have there three constituents, "Scott," "is," and "the author

of *Waverley*." That, of course, is the sort of way you might think of analyzing. You might admit that "the author of *Waverley*" was complex and could be further cut up, but you might think the proposition could be split into those three bits to begin with. That is an entire mistake. "The author of *Waverley*" is not a constituent of the proposition at all. There is no constituent really there corresponding to the descriptive phrase. I will try to prove that to you now.

The first and most obvious reason is that you can have significant propositions denying the existence of "the so-and-so." "The unicorn does not exist." "The greatest finite number does not exist." Propositions of that sort are perfectly significant, are perfectly sober, true, decent propositions, and that could not possibly be the case if the unicorn were a constituent of the proposition, because plainly it could not be a constituent as long as there were not any unicorns. Because the constituents of propositions, of course, are the same as the constituents of the corresponding facts, and since it is a fact that the unicorn does not exist, it is perfectly clear that the unicorn is not a constituent of that fact, because if there were any fact of which the unicorn was a constituent, there would be a unicorn, and it would not be true that it did not exist. That applies in this case of descriptions particularly. Now since it is possible for "the so-and-so" not to exist and yet for propositions in which "the so-and-so" occurs to be significant and even true, we must try to see what is meant by saying that the so-and-so does exist.

The occurrence of tense in verbs is an exceedingly annoying vulgarity due to our preoccupation with practical affairs. It would be much more agreeable if they had no tense, as I believe is the case in Chinese, but I do not know Chinese. You ought to be able to say "Socrates exists in the past," "Socrates exists in the present" or "Socrates

exists in the future," or simply "Socrates exists," without any implication of tense, but language does not allow that, unfortunately. Nevertheless, I am going to use language in this tenseless way: when I say "The so-and-so exists," I am not going to mean that it exists in the present or in the past or in the future, but simply that it exists, without implying anything involving tense.

"The author of *Waverley* exists": there are two things required for that. First of all, what is "the author of *Waverley*"? It is the person who wrote *Waverley,* i. e., we are coming now to this, that you have a propositional function involved, viz., "x writes *Waverley,*" and the author of *Waverley* is the person who writes *Waverley,* and in order that the person who writes *Waverley* may exist, it is necessary that this propositional function should have two properties:

1. It must be true for *at least* one x.
2. It must be true for *at most* one x.

If nobody had ever written *Waverley* the author could not exist, and if two people had written it, *the* author could not exist. So that you want these two properties, the one that it is true for at least one x, and the other that it is true for at most one x, both of which are required for existence.

The property of being true for at least one x is the one we dealt with last time: what I expressed by saying that the propositional function is *possible.* Then we come on to the second condition, that it is true for at most one x, and that you can express in this way: "If x and y wrote *Waverley,* then x is identical with y, whatever x and y may be." That says that at most one wrote it. It does not say that anybody wrote *Waverley* at all, because if nobody had written it, that statement would still be true. It only says that at most one person wrote it.

The first of these conditions for existence fails in the case of the unicorn, and the second in the case of the inhabitant of London.

We can put these two conditions together and get a portmanteau expression including the meaning of both. You can reduce them both down to this, that: "('x wrote *Waverley*' is equivalent to 'x is c' whatever x may be) is possible in respect of c." That is as simple, I think, as you can make the statement.

You see that means to say that there is some entity c, we may not know what it is, which is such that when x is c, it is true that x wrote *Waverley*, and when x is not c, it is not true that x wrote *Waverley*, which amounts to saying that c is the only person who wrote *Waverley*; and I say there is a value of c which makes that true. So that this whole expression, which is a propositional function about c, is *possible* in respect of c (in the sense explained last time).

That is what I mean when I say that the author of *Waverley* exists. When I say "the author of *Waverley* exists," I mean that there is an entity c such that "x wrote *Waverley*" is true when x is c, and is false when x is not c. "The author of *Waverley*" as a constituent has quite disappeared there, so that when I say "The author of *Waverley* exists" I am not saying anything about the author of *Waverley*. You have instead this elaborate to-do with propositional functions, and "the author of *Waverley*" has disappeared. That is why it is possible to say significantly "The author of *Waverley* did not exist." It would not be possible if "the author of *Waverley*" were a constituent of propositions in whose verbal expression this descriptive phrase occurs.

The fact that you can discuss the proposition "God exists" is a proof that "God," as used in that proposition,

is a description and not a name. If "God" were a name, no question as to existence could arise.

I have now defined what I mean by saying that a thing described exists. I have still to explain what I mean by saying that a thing described has a certain property. Supposing you want to say "The author of *Waverley* was human," that will be represented thus: "('*x* wrote *Waverley*' is equivalent to '*x* is *c*' whatever *x* may be, and *c* is human) is possible with respect to *c*."

You will observe that what we gave before as the meaning of "The author of *Waverley* exists" is part of this proposition. It is part of any proposition in which "the author of *Waverley*" has what I call a "primary occurrence." When I speak of a "primary occurrence I mean that you are not having a proposition about the author of *Waverley* occurring as a part of some larger proposition, such as "I believe that the author of *Waverley* was human" or "I believe that the author of *Waverley* exists." When it is a primary occurrence, i. e., when the proposition concerning it is not just part of a larger proposition, the phrase which we defined as the meaning of "The author of *Waverley* exists" will be part of that proposition. If I say the author of *Waverley* was human, or a poet, or a Scotsman, or whatever I say about the author of *Waverley* in the way of a primary occurrence, always this statement of his existence is part of the proposition. In that sense all these propositions that I make about the author of *Waverley* imply that the author of *Waverley* exists. So that any statement in which a description has a primary occurrence implies that the object described exists. If I say "The present King of France is bald," that implies that the present King of France exists. If I say, "The present King of France has a fine head of hair," that also implies that the present King of France exists. Therefore unless you understand how a

proposition containing a description is to be denied, you will come to the conclusion that it is not true either that the present King of France is bald or that he is not bald, because if you were to enumerate all the things that are bald you would not find him there, and if you were to enumerate all the things that are not bald, you would not find him there either. The only suggestion I have found for dealing with that on conventional lines is to suppose that he wears a wig. You can only avoid the hypothesis that he wears a wig by observing that the denial of the proposition "The present King of France is bald" will not be "The present King of France is not bald," if you mean by that "There is such a person as the King of France and that person is not bald." The reason of this is that when you state that the present King of France is bald you say "There is a *c* such that *c* is now King of France and *c* is bald" and the denial is not "There is a *c* such that *c* is now King of France and *c* is not bald." It is more complicated. It is: "Either there is not a *c* such that *c* is now King of France, or, if there is such a *c*, then *c* is not bald." Therefore you see that, if you want to deny the proposition "The present King of France is bald," you can do it by denying that he exists, instead of by denying that he is bald. In order to deny this statement that the present King of France is bald, which is a statement consisting of two parts, you can proceed by denying either part. You can deny the one part, which would lead you to suppose that the present King of France exists but is not bald, or the other part, which will lead you to the denial that the present King of France exists; and either of those two denials will lead you to the falsehood of the proposition "The present King of France is bald." When you say "Scott is human" there is no possibility of a double denial. The only way you can deny "Scott is human" is by saying

"Scott is not human." But where a descriptive phrase occurs, you do have the double possibility of denial.

It is of the utmost importance to realize that "the so-and-so" does not occur in the analysis of propositions in whose verbal expression it occurs, that when I say "The author of *Waverley* is human," "the author of *Waverley*" is not the subject of that proposition, in the sort of way that Scott would be if I said "Scott is human," using "Scott" as a name. I cannot emphasize sufficiently how important this point is, and how much error you get into metaphysics if you do not realize that when I say "The author of *Waverley* is human" that is not a proposition of the same form as "Scott is human." It does not contain a constituent "the author of *Waverley*." The importance of that is very great for many reasons, and one of them is this question of existence. As I pointed out to you last time, there is a vast amount of philosophy that rests upon the notion that existence is, so to speak, a property that you can attribute to things, and that the things that exist have the property of existence and the things that do not exist do not. That is rubbish, whether you take kinds of things, or individual things described. When I say, e. g., "Homer existed," I am meaning by "Homer" some description, say "the author of the Homeric poems," and I am asserting that those poems were written by one man, which is a very doubtful proposition; but if you could get hold of the actual person who did actually write those poems (supposing there was such a person), to say of him that he existed would be uttering nonsense, not a falsehood but nonsense, because it is only of persons described that it can be significantly said that they exist. Last time I pointed out the fallacy in saying "Men exist, Socrates is a man, therefore Socrates exists." When I say "Homer exists, this is Homer, therefore this exists," that is a fallacy of the same sort. It is an entire mistake to argue: "This is the author

of the Homeric poems and the author of the Homeric poems exists, therefore this exists." It is only where a propositional function comes in that existence may be significantly asserted. You can assert "The so-and-so exists," meaning that there is just one *c* which has those properties, but when you get hold of a *c* that has them, you cannot say of this *c* that it exists, because that is nonsense: it is not false, but it has no meaning at all.

So the individuals that there are in the world do not exist, or rather it is nonsense to say that they exist and nonsense to say that they do not exist. It is not a thing you can say when you have named them, but only when you have described them. When you say "Homer exists," you mean "Homer" is a description which applies to something. A description when it is fully stated is always of the form "the so-and-so."

The sort of things that are like these descriptions in that they occur in words in a proposition, but are not in actual fact constituents of the proposition rightly analyzed, things of that sort I call "incomplete symbols." There are a great many sorts of incomplete symbols in logic, and they are sources of a great deal of confusion and false philosophy, because people get misled by grammar. You think that the proposition "Scott is mortal" and the proposition "The author of *Waverley* is mortal" are of the same form. You think that they are both simple propositions attributing a predicate to a subject. That is an entire delusion: one of them is (or rather might be) and one of them is not. These things, like "the author of *Waverley*," which I call incomplete symbols, are things that have absolutely no meaning whatsoever in isolation but merely acquire a meaning in a context. "Scott" taken as a name has a meaning all by itself. It stands for a certain person, and there it is. But "the author of *Waverley*" is not a name, and does not all by itself mean anything at all, because when it is rightly

used in propositions, those propositions do not contain any constituent corresponding to it.

There are a great many other sorts of incomplete symbols besides descriptions. These are classes, which I shall speak of next time, and relations taken in extension, and so on. Such aggregations of symbols are really the same thing as what I call "logical fictions," and they embrace practically all the familiar objects of daily life: tables, chairs, Piccadilly, Socrates, and so on. Most of them are either classes, or series, or series of classes. In any case they are all incomplete symbols, i. e, they are aggregations that only have a meaning in use and do not have any meaning in themselves.

It is important, if you want to understand the analysis of the world, or the analysis of facts, or if you want to have any idea what there really is in the world, to realize how much of what there is in phraseology is of the nature of incomplete symbols. You can see that very easily in the case of "the author of *Waverley*" because "the author of *Waverley*" does not stand simply for Scott, nor for anything else. If it stood for Scott, "Scott is the author of *Waverley*" would be the same proposition as "Scott is Scott," which it is not, since George IV wished to know the truth of the one and did not wish to know the truth of the other. If "the author of *Waverley*" stood for anything other than Scott, "Scott is the author of *Waverley*" would be false, which it is not. Hence you have to conclude that "the author of *Waverley*" does not, in isolation, really stand for anything at all; and that is the characteristic of incomplete symbols.

[TO BE CONCLUDED.]

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